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Deeper Discussions in Math Add Up

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Getting students to think—and talk—like mathematicians.

During English language arts lessons, I liked to tell my elementary school students, "We are going to be authors today and write about what we have learned," or, "We are going to read like authors and question the choices the author made in our book." These prompts empower students, who usually jump right into their role as authors. They engage in the reading and writing process with a critical eye, questioning both the author and each other.

For a long time, elementary school teachers have urged students to see themselves as active learners in ELA, owning their role in the learning process by reading and writing as authors do. But for some reason, many of these same teachers don't think students are ready for this role when learning math—they expect students to learn what they are shown instead of creating and exploring their own understandings.

In math class, we often boil concepts down to their most basic sequence of steps so students can easily arrive at the right answer. We show students how to do a problem the right way and expect them to follow suit, without a lot of back and forth. *This is a missed opportunity*.

I had a math coach once tell me: "The people who do the talking in any lesson are the ones doing the learning." If we start asking elementary students to step up and own their math learning from the very beginning, as we do in ELA, students will better understand concepts and have fewer problems with retention and application. A research brief on discussion by the National Council of Teachers of Mathematics explains that engaging students in discourse during math class can:

- increase student learning,
- motivate students,
- support teachers in understanding and assessing student thinking, and
- shift the mathematical authority from teacher (or textbook) to community (Cirillo, 2013).

It is time to make a change and start our math lessons by telling students, "Today we are going to think—and *talk*—like mathematicians."

First Grade: Counting On and Counting Back

While supporting instruction as a math coach one day, I challenged a group of 1st grade students to "think and talk like mathematicians" during a lesson on using the counting on and counting back strategies for subtraction. These students had used counting on to add and counting back to subtract, but today I wanted them to build on this understanding and take their understanding of counting on a step further.

I opened with the question, "How can we find the answer to 9 − 7?" Building off the previous day's instruction, the students immediately shared a few possibilities. Student responses included: "Use a picture showing a set of 9 and then take 7 away—whatever is left will be the answer." One student qualified an answer with, "Because I did it in my head, so I know." A third student explained that we could count back 7 starting at 9.

Figure 1. Number bond showing the parts, 7 and 2, that make up the whole, 9

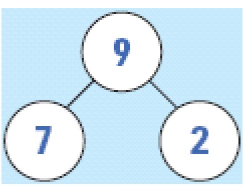


Figure 2. Number path showing the sequence of numbers 1–10

1	2	3	4	5	6	7	8	9	10

Figure 3a. Tape diagram showing a rectangle divided into 6 equal parts that are then divided in half

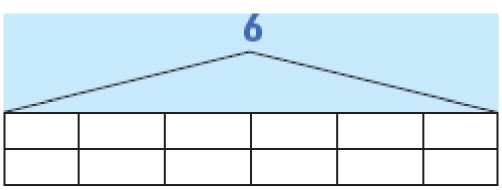
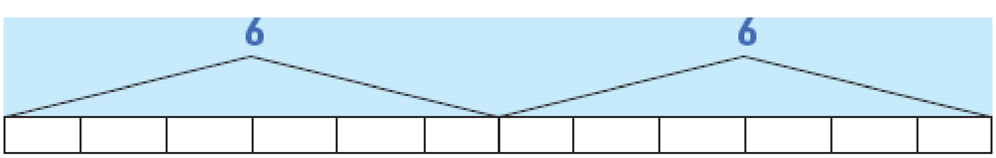


Figure 3b. Tape diagram twice as long as Figure 3a, showing two sets of 6 half-units next to each other



Since my goal was for the students to develop their own understanding, I then asked an open-ended question: "Is there a way we could use *counting on*, or addition, to solve this subtraction problem?" This time there was a long pause. I added a visual (see fig. 1) to the board to see if it might help.

After thinking about the number bond, which shows how numbers are split or combined, two students raised their hand. One explained, "9 − 7 = 2 has the same parts and whole as 7 + 2 = 9." The next student said, "When we counted back 7 starting at 9 we had 2 left, and if we count on from 7 up to 9 we have to count on 2. Both of those 2's are that part (pointing to the 2 in the bottom right corner) in the number bond."

One student then raised his hand and challenged the group. "Adding and subtracting are different though," he said. "You can't add to subtract!" While I was tempted to jump in, I forced myself to pause and see if one of the students would respond. Soon another boy replied, "But addition and subtraction kind of go together." I asked this student to say more about what he meant. He went on to explain, "They just tell us how the parts and whole go together or get taken apart." A third student jumped in: "Yeah, the 2 we had left when we counted back is the same part you add when we counted on."

This moment was so powerful. It was amazing to see my students questioning each other's thinking and not just internalizing a sequence of steps. Opportunities for students to share their thinking with, ask questions of, and respond to the thinking of other students are key to deeper discussions in the math classroom (NCTM, 2014).

Another student took the discussion to the next level when she added, "I like the idea of counting on from 7 because it is closer to 9, and I won't lose track and make as many mistakes." The conversation now shifted from, "*Can we* use counting on when we subtract?" to "*Why might we* use counting on when we subtract?" I added a number path (see fig. 2) to the board, and our discussion about the comparative merits of counting on and counting back continued. We discussed efficiency and how much we would need to count using each strategy in different scenarios, knowing that less counting means moving faster and possibly making fewer mistakes.

For these young students, this discussion provided access to new content and deepened their conceptual understanding. The students used what they had previously learned—the strategies of counting on and counting back—and applied them to a new situation. As the teacher, I never told the students what I wanted them to learn. I simply facilitated the conversation, using a few carefully planned questions to guide them toward discovering a new way to use a familiar strategy. Their interactions with me and, more important, with each other led them to new understandings.

Fifth Grade: Solving Equations

Sometimes deeper discussion not only helps students learn new content, but can also fill in gaps in content students have already "learned." In 5th grade, for example, students practice dividing by a fraction. A common shortcut teachers use to help students is *keep, change, flip*, which teaches students to *keep* the first number the same, *change* the sign from division to multiplication, and *flip* the numerator and denominator in the second number.

During a co-taught lesson, the classroom teacher asked me to meet with a group of students who were struggling with dividing a whole number by a fraction. I discovered that though these students knew how to find the right answer for this calculation using the shortcut, they didn't understand what the shortcut meant and why it worked. The students needed to engage in a deeper discussion to make sense of the process they were using and really grasp the concept.

I began this conversation by showing the students the following calculation and asking them if I had the correct answer:

$$6 \div 1/2 = 6 \times 2 = 12$$

Since all the students agreed that I got it right, I then asked them to "think and talk like mathematicians" and to draw me a picture that proved my answer was correct. But now the students were clearly lost. They had no idea where to start. While planning this lesson, I anticipated this moment and created two pictures (see figures 3a and 3b). I decided that instead of walking them through the modeling process and explaining my work, I would ask the students to explain what they noticed about the models.

The students in my group listed things like, "Both pictures have 6's," "There are two sets of 6 in each picture," "One has one row of 12, and the other has two rows of 6," and "Both pictures have 12 parts all together."

After this brainstorm was complete, I asked, "How are these models related to the equation I showed you first?" There was a significant pause as the students considered this question. Then, one student tentatively offered that the first picture was like the first part of the work, 6 ÷ 1/2, and the second picture was like the middle part, 6 × 2. Another student added, "Both pictures show 12, like the answer."

I knew the students were close to being able to explain the relationship, but they were not quite there yet. So, I asked one more question: "How do these pictures prove why the *keep, change, flip* shortcut works?" Again, the students paused to consider the two pictures and the calculation. It felt like an eternity as I waited for them to process, but my patience paid off. One student said, "Wait! I get it! The first picture shows it actually divided in half—each one of the pieces is divided in half. The second picture is just like if we take the two rows of pieces and line them up next to each other. That picture is the same as drawing 6 × 2. That's why it works!"

Throughout this discussion I kept pressing students to explain how or why they knew something. These follow-up questions are powerful because they are equally relevant whether a student's thinking is right or wrong. When follow-up questions are asked consistently, it creates a culture in which students know that being able to explain their thinking is important and expected. This process of justifying can often bring clarity to how students arrive at the correct answer or help students discover the flaw in their own reasoning if they are not correct (Wills & Rothermal Rawding, 2012).

After this epiphany, the group moved on to practice drawing 3 ÷ 1/2, 5 ÷ 1/3, and 4 ÷ 1/4. Each time, the students had to explain their thinking to a partner before they could write the equation to show their calculations. At the end of the lesson, one student pointed out to me that even though she could get the right answer before, "It feels better to know where it came from than to just believe it is right."

Bottom Line: Facilitating a Discussion Is Key

Students and teachers often approach math with a single goal in mind: to get the right answer. But teachers must resist the urge to simply teach a procedure. Instead, they should make a conscious decision to facilitate a discussion that leads to deeper student learning. This shift in classroom culture can be as simple as changing your practice from reviewing the answers to questions completed during independent practice to asking students *which question* they found the most challenging.

Math instruction needs to help students use what they know to make sense of concepts, look at problems though multiple lenses, and understand how to explain the *why* and *how* of the thinking process. Instructional planning needs to focus less on what teachers want to tell the students and more on well-crafted sequences of questions. A *focused pattern* of questions includes questions that require students to identify and use basic information, that challenge student thinking, that make connections between abstract representations and models that convey meaning, and that ask students to provide explanations and justifications (NCTM, 2014).

Mathematicians approach questions by thinking to themselves, "How can I use what I know in this new situation?" instead of "I don't know. No one taught me how to do this yet." It is time for our students to embrace this practice, too, and to do so aloud and in conversation with their peers and teachers. It is time for students to discover the right answers through *exploration* instead of waiting to be told them.

References

Cirillo, M. (2013). What does research say the benefits of discussion in the mathematics classroom are? [PDF]. NCTM.

National Council of Teachers of Mathematics (NCTM). (2014). Principals to actions ensuring mathematical success for all. Reston, VA.

Wills, T., & Rothermal Rawding, M. (2012). Discourse: Simple moves that work. *Mathematics Teaching in the Middle School, 18*(1), 47–51.

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